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Effects of varying viscosity and thermal conductivity on steady MHD free convective flow and heat transfer along an isothermal plate with internal heat generation

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Abstract

Purpose – Physical properties of a viscous fluid, e.g. viscosity and thermal conductivity change with temperature and in most of the studies concerned with natural convection, generally, the simultaneous effect of temperature dependent viscosity, thermal conductivity have been neglected. Hence, the purpose of this paper is to investigate the simultaneous effects of varying viscosity and thermal conductivity on free convection flow of a viscous incompressible electrically conducting fluid and heat transfer along an isothermal vertical non-conducting plate in the presence of exponentially varying internal heat-generation and uniform transverse magnetic field.

Design/methodology/approach – The governing equations of motion and energy are transformed into ordinary differential equations using similarity transformation. The resulting boundary valued, coupled and non-linear differential equations are converted into system of linear differential equations and solved using Runge-Kutta fourth order technique along with shooting method.

Findings – It was found that: fluid velocity decreases with the increase in magnetic parameter or Prandtl number; fluid temperature increases with the increase in magnetic parameter; velocity and temperature profiles increase due to increase in heat generation parameter; varying viscosity and thermal conductivity modifies the flow and heat transfer characteristic; and skin-friction and heat transfer are affected by simultaneous change in viscosity and thermal conductivity in presence/absence of exponentially varying heat generation.

Research limitations/implications – The present study is applicable to an incompressible viscous fluid flow and heat transfer with linearly varying viscosity and thermal conductivity.

Originality/value – This paper provides useful information on the physical properties of a viscous fluid with regard to viscosity and thermal conductivity change with temperature.

Keywords Convection, Viscosity, Thermal conductivity, Heat, Friction

Paper type Research paper

Nomenclature

B_o = magnetic field intensity
C_f = skin-friction coefficient
C_p = specific heat at constant pressure
f = dimensionless stream function

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g	= acceleration due to gravity of the earth
Gr	= Grashof number $\{=g\beta(T_w - T_\infty)x^3/\nu^2\}$
M	= magnetic parameter $\{=(\sigma B_0^2 x^2)/(\mu)\cdot(Gr/4)^{-(1/2)}\}$
Nu	= Nusselt number
Pr	= Prandtl number $(=\mu C_p/\kappa)$
Q	= volumetric rate of heat generation $\{=\kappa(T_w - T_\infty/x^2)\cdot(Gr/4)^{1/2}e^{-\eta}\}$
S	= heat generation parameter
T	= temperature of the fluid
T_w	= temperature of the plate
T_∞	= temperature of fluid far from plate
u, v	= velocity components along x- and y-directions
x, y	= Cartesian coordinates

Greek letters

σ	= electrical conductivity
β	= coefficient of thermal expansion
ψ	= stream function
η	= similarity variable
ε, γ	= thermal conductivity and viscosity parameter
θ	= dimensionless temperature $\{=(T - T_\infty)/(T_w - T_\infty)\}$
μ^*, μ	= variable viscosity, coefficient of viscosity, respectively
κ^*, κ	= variable thermal conductivity, thermal conductivity, respectively
ν, ρ	= kinematic viscosity $(=\mu/\rho)$, fluid density, respectively

Superscript

$'$	= differentiation with respect to η

1. Introduction

The natural convection process due to internal heat generation is present in various physical phenomena such as fire engineering, combustion modeling, nuclear energy, heat exchangers, petroleum reservoir, etc. Hydromagnetic flows have also become important due to industrial applications. Ostrach (1952) presented the similarity solution of natural convection along vertical isothermal plate. Gebhart (1962) used perturbation technique to analyse the effect of dissipation on viscous flow. Soundalgekar (1976) studied natural convection flow along vertical porous plate with suction and viscous dissipation. Carey and Mollendorf (1978) studied the effect of temperature dependent viscosity on free convective fluid flow. Joshi and Gebhart (1981) observed the effect of pressure stress work and viscous dissipation in some natural convection flow: isothermal, uniform flux and plumes. Takhar *et al.* (1996) analysed the free convection heat transfer in the presence of transverse magnetic field of viscous fluid along a semi-infinite vertical plate. Crepeau and Clarksean (1997) discussed similarity solution of natural convection with internal heat generation, which decays exponentially. Chamkha and Issa (2000) obtained the similarity solution of hydromagnetic flow along permeable flat surface with heat generation/absorption and thermophoresis effect. Chamkha and Khaled (2000) observed the hydromagnetic natural convection from a permeable surface embedded in fluid. Chamkha and Khaled (2001) obtained similarity solution of natural convection on an inclined plate with internal heat generation/absorption in presence of transverse magnetic filed. Hossain *et al.* (2001) observed the effect of radiation on free convective flow on vertical porous plate with variable viscosity. Hossain *et al.* (2001) analysed the effect of temperature dependent viscosity and thermal conductivity in natural convection along a truncated cone. Chen (2004) studied MHD flow by natural convection on an inclined plate with power law variation of temperature and concentration. Soundalgekar *et al.* (2004) presented the results due to the effects of variable viscosity along a moving plate with variable surface temperature. Seddeek and Salem (2005) discussed the effect of variable viscosity and thermal diffusivity on mixed convection flow along vertical stretching sheet.

It is known that physical properties, e.g. viscosity, thermal conductivity change with temperature. In most of the studies concerned with natural convection, generally, the simultaneous effects of temperature dependent viscosity, thermal conductivity have been neglected. When these effects are included, the flow and heat transfer characteristic may change considerably. Hence, the present study intends to investigate the effects of varying viscosity and thermal conductivity on free convection flow of a viscous incompressible electrically conducting fluid and heat transfer along an isothermal vertical non-conducting plate in the presence of internal heat-generation and uniform transverse magnetic field.

2. Formulation of the problem

Consider steady laminar 2D free convection flow of a viscous incompressible fluid along a vertical non-conducting plate kept at constant temperature T_w, and the fluid has internal volumetric rate of heat generation Q. The x-axis is taken along the plate and y-axis is normal to the plate. Magnetic field of uniform intensity B_o is applied in y-direction. It is assumed that the external field is zero, also electrical field due to polarization of charges and Hall effect are neglected. Incorporating the Boussinesq approximation within the boundary layer, the governing equations of continuity, momentum and energy (Schlichting, 1968; Bansal, 1994), neglecting the viscous dissipation effect, respectively, are given by:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu^*\frac{\partial u}{\partial y}\right) + g\beta(T - T_\infty) - \frac{\sigma B_o^2(x)}{\rho}u, \tag{2}$$

$$\rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial y}\left(\kappa^*\frac{\partial T}{\partial y}\right) + Q. \tag{3}$$

The boundary conditions are:

$$y = 0: \quad u = 0, v = 0, T = T_w$$
$$y \to \infty : u = 0, T = T_\infty. \tag{4}$$

The variable viscosity (Carey and Mollendorf, 1978) and thermal conductivity (Seddeek and Salem, 2005) are considered to vary linearly with temperature as given below, respectively:

$$\mu^* = \mu\left[1 + \gamma\left(\theta - \frac{1}{2}\right)\right], \tag{5}$$

$$\kappa^* = \kappa[1 + \varepsilon\theta], \tag{6}$$

where γ and ε are perturbation parameters.

3. Method of solution

Introducing the stream function $\psi\ (x,y)$ such that:

$$u = \frac{\partial\psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial\psi}{\partial x}, \tag{7}$$

where:

$$\psi(x,y) = 4\nu f(\eta)\left(\frac{Gr}{4}\right)^{1/4},$$

and the similarity variable:

$$\eta = \frac{y}{x}\left(\frac{Gr}{4}\right)^{1/4}. \quad (8)$$

Following Crepeau and Clarksean (1997), the volumetric rate of heat generation is taken as given below:

$$Q = S\left\{\kappa\left(\frac{T_w - T_\infty}{x^2}\right)\left(\frac{Gr}{4}\right)^{1/2} e^{-\eta}\right\}, \quad (9)$$

where Gr is the Grashof number and S is heat generation parameter.

It is observed that the equation (1) is identically satisfied by $\psi\ (x,y)$. Substituting equations (8) and (9) into the equations (2) and (3), along with the equations (5) and (6), the resulting non-linear ordinary differential equations are:

$$\left\{1 + \gamma\left(\theta - \frac{1}{2}\right)\right\}f''' + \gamma\theta' f'' - 2f'^2 + 3ff'' + \theta - Mf' = 0, \quad (10)$$

and:

$$(1 + \varepsilon\theta)\theta'' + \varepsilon\theta'^2 + 3Pr\theta' f + Se^{-\eta} = 0, \quad (11)$$

where:

$$M\left\{= \frac{\sigma B_o^2 x^2}{\mu}\left(\frac{Gr}{4}\right)^{-(1/2)}\right\},$$

is magnetic parameter, which represents the importance of magnetic field. The magnetic field intensity B_o must be proportional to $x^{-(1/4)}$, to eliminate the dependency of M on x (Chen, 2004).

The boundary conditions are reduced to:

$$f(0) = 0, f'(0) = 0, f'(\infty) = 0,\ \theta(0) = 1\ \text{and}\ \theta(\infty) = 0. \quad (12)$$

The governing boundary layer equations (10) and (11) are coupled non-linear differential equations and solved using Runge-Kutta fourth order technique along with double shooting technique (Conte and Boor, 1981). The solution methodology being, first the higher order non-linear coupled differential equations (10) and (11) are decomposed into system of first order differential equations as given below:

$$\frac{\partial f}{\partial \eta} = U = f_1(\eta, f, U, V, \theta, W), \quad f(0) = 0, \quad (13)$$

$$\frac{\partial U}{\partial \eta} = V = f_2(\eta, f, U, V, \theta, W), \quad U(0) = 0, \quad (14)$$

$$\frac{\partial V}{\partial \eta} = f_3(\eta, f, U, V, \theta, W) = \frac{(-\gamma WV + 2U^2 - 3fV - \theta + MU)}{\{1 + \gamma(\theta - 1/2)\}}, \quad V(0) = ? \quad (15)$$

$$\frac{\partial \theta}{\partial \eta} = W = f_4(\eta, f, U, V, \theta, W), \quad \theta(0) = 1, \quad (16)$$

$$\frac{\partial W}{\partial \eta} = f_5(\eta, f, U, V, \theta, W) = \frac{[-\varepsilon W^2 - 3PrWf - Se^{-\eta}]}{(1 + \varepsilon\theta), W(0) = ?} \quad (17)$$

Now, with the help of shooting technique $V(0)$ and $W(0)$ are approximated, as explained by Conte and Boor (1981). Hence, the system of equations (13)-(17) is reduced to a system of initial value problem which is solved using Runge-Kutta technique. While shooting, to get the value of $V(0)$ and $W(0)$, care has been taken to shoot in steps and the shoots are improved in stages. While solving the system of equations, the step size is kept 0.005.

4. Skin-friction coefficient

Skin-friction coefficient at the plate is given by:

$$C_f = \left(1 + \frac{\gamma}{2}\right)(Gr)^{3/4} f''(0). \quad (18)$$

5. Nusselt number

The rate of heat transfer in terms of the Nusselt number at the plate is given by:

$$Nu = -(1 + \varepsilon)(Gr)^{1/4} \theta'(0). \quad (19)$$

6. Particular cases

- In the absence of magnetic field, heat generation and constant thermal conductivity, i.e. $M = 0$, $S = 0$ and $\varepsilon = 0$; the results of present paper are reduced to those obtained by Carey and Mollendorf (1978) and Hossain *et al.* (2001).
- In the absence of magnetic field and taking constant viscosity and thermal conductivity, i.e. $M = 0$, $\gamma = 0$ and $\varepsilon = 0$; the results of present paper are reduced to those obtained by Crepeau and Clarksean (1997) and Chamkha and Khaled (2001).

7. Results and discussion

It is observed from Table I that the numerical values of $f''(0)$ and $\theta'(0)$ for $M = 0.0$, $\varepsilon = 0.0$ and $S = 0$ obtained in the present paper are in good agreement with those obtained by Hossain *et al.* (2001) and Carey and Mollendorf (1978).

It is seen from Table II that the numerical results of $\theta'(0)$ of present paper are in good agreement with those obtained by Chamkha and Khaled (2001) and Crepeau and Clarksean (1997) when $M = 0.0$, $\gamma = 0.0$ and $\varepsilon = 0.0$.

The numerical values of $f''(0)$ and $\theta'(0)$ are presented in Table III. It is observed that with the increase in magnetic field intensity, $f''(0)$ and $-\theta'(0)$ decrease, while with the increase in Prandtl number, $f''(0)$ decreases but $-\theta'(0)$ increases.

From Table IV, when heat generation is absent (i.e. $S = 0.0$), with the increase in γ, the numerical value of $f''(0)$ and $-\theta'(0)$ decrease for different values of Prandtl number. With the increase of ε, $f''(0)$ increases while $-\theta'(0)$ decreases. In the presence of heat generation (i.e. $S = 1.0$); with increase in γ, both $f''(0)$ and $-\theta'(0)$ decrease. For $Pr = 1$ and 10 with the increase in ε; $f''(0)$ increases, while it decreases at $Pr = 0.1$ (<1). The values of $-\theta'(0)$ increase with the increase in ε when $Pr = 0.1$ and 1.0, while it decreases when $Pr = 10$ (>1). Hence, behavioural changes in fluid flow are observed with respect to different parameters.

Figures 1 and 2 show the velocity profile at different values of magnetic field intensity for different Prandtl number when $S = 0.0$ and $S = 1.0$ (i.e. in absence and presence of Q), respectively. The velocity profiles decrease with increase in magnetic field intensity because presence of magnetic field introduces Lorentz force, which acts against the flow. The velocity profiles at low-Prandtl number are higher and magnetic field affects more the flow of fluid. It is interesting to note that the maximum of velocity profile is achieved approximately at same value of η for given Prandtl number, even

	Hossain *et al.* (2001)		Carey and Mollendorf (1978)		Present work	
γ	$f''(0)$	$\theta'(0)$	$f''(0)$	$\theta'(0)$	$f''(0)$	$\theta'(0)$
0.0	0.6421	−0.5671	0.6422	−0.5671	0.642187	−0.567145
0.8	0.5050	−0.5469	0.5050	−0.5469	0.505014	−0.546940
1.6	0.4222	−0.5281	0.4233	−0.5315	0.422341	−0.531531
−1.6	2.0411	−0.6514	2.0416	−0.6514	2.041617	−0.651368

Table I. Values of $f''(0)$ and $\theta'(0)$ for different values of γ when $Pr = 1.0$ and $S = 0$ are compared with results obtained by Hossain *et al.* (2001) and Carey and Mollendorf (1978)

	Chamkha and Khaled (2001)		Crepeau and Clarksean (1997)		Present work	
	Without Q	With Q	Without Q	With Q	Without Q	With Q
Pr	$\theta'(0)$	$\theta'(0)$	$\theta'(0)$	$\theta'(0)$	$\theta'(0)$	$\theta'(0)$
0.1	−0.2119	0.5656	−0.2302	0.5425	−0.230136	0.542484
1.0	−0.5646	0.005788	−0.5671	0.005786	−0.567145	0.0058026
10	−1.1720	−0.8027	−1.169	−0.7963	−1.166270	−0.796247

Table II. Values of $\theta'(0)$ for different values of Pr are compared with the results obtained by Chamkha and Khaled (2001) and Crepeau and Clarksean (1997)

	$M = 0.0$		$M = 0.5$		$M = 1.0$	
Pr	$f''(0)$	$\theta'(0)$	$f''(0)$	$\theta'(0)$	$f''(0)$	$\theta'(0)$
S = 0.0						
0.1	0.8591426	−0.230136	0.7648784	−0.215228	0.6894158	−0.2016967
1.0	0.6421876	−0.567145	0.5882680	−0.534512	0.5452539	−0.506551
10	0.4191727	−1.169270	0.3925033	−1.115481	0.3734609	−1.075662
S = 1.0						
0.1	1.0679157	0.5424841	0.9593559	0.5627266	0.8690657	0.5818464
1.0	0.7656694	0.0058026	0.7032612	0.0488207	0.6521948	0.0868764
10	0.4767093	−0.796247	0.4464109	−0.728097	0.4239770	−0.675245

Table III. Values of $f''(0)$ and $\theta'(0)$ for different values of M when $\gamma = 0.0$ and $\varepsilon = 0.0$

Table IV.
Values of $f''(0)$ and $\theta'(0)$ for different values of γ, ε and Pr when $M = 0.5$

	$\gamma = -0.4$		$\gamma = 0.0$		$\gamma = 0.4$	
	$f''(0)$	$\theta'(0)$	$f''(0)$	$\theta'(0)$	$f''(0)$	$\theta'(0)$
S = 0						
Pr = 0.1.0						
$\varepsilon = 0.0$	0.87811392	−0.217915	0.7648784	−0.215228	0.68303101	−0.212897
$\varepsilon = 0.1$	0.88379286	−0.205627	0.7705048	−0.203140	0.68854294	−0.200981
$\varepsilon = 0.3$	0.89353192	−0.186111	0.7801610	−0.183946	0.69801017	−0.182062
Pr = 1.0						
$\varepsilon = 0.0$	0.6876498	−0.546067	0.5882680	−0.534512	0.5178665	−0.524895
$\varepsilon = 0.1$	0.6953315	−0.517629	0.5954330	−0.506705	0.5245969	−0.497603
$\varepsilon = 0.3$	0.7090450	−0.472526	0.6082024	−0.462632	0.5365781	−0.454368
Pr = 10						
$\varepsilon = 0.0$	0.46518397	−1.145613	0.3925033	−1.115481	0.34174401	−1.091303
$\varepsilon = 0.1$	0.47182826	−1.088938	0.3984941	−1.060151	0.34723953	−1.037029
$\varepsilon = 0.3$	0.48394139	−0.999417	0.4093971	−0.972831	0.35722888	−0.951444
S = 1.0						
Pr = 0.1						
$\varepsilon = 0.0$	1.0517857	0.5586113	0.9593559	0.5627665	0.8867136	0.5664639
$\varepsilon = 0.1$	1.0462035	0.5064377	0.9517817	0.5101068	0.8781571	0.5134370
$\varepsilon = 0.3$	1.0376193	0.4251676	0.9464512	0.4426721	0.8713283	0.4445602
Pr = 1.0						
$\varepsilon = 0.0$	0.80394711	0.0319877	0.7032612	0.0488207	0.63040227	0.0631901
$\varepsilon = 0.1$	0.80624796	0.0133250	0.7050232	0.0288929	0.63179185	0.0421844
$\varepsilon = 0.3$	0.81072040	−0.014822	0.7085714	−0.001228	0.63469290	0.0103831
Pr = 10						
$\varepsilon = 0.0$	0.52489892	−0.765848	0.4464109	−0.728097	0.39134120	−0.697158
$\varepsilon = 0.1$	0.52954051	−0.740608	0.4506255	−0.704925	0.39522163	−0.675685
$\varepsilon = 0.3$	0.53811241	−0.699643	0.4583865	−0.667271	0.40235418	−0.640751

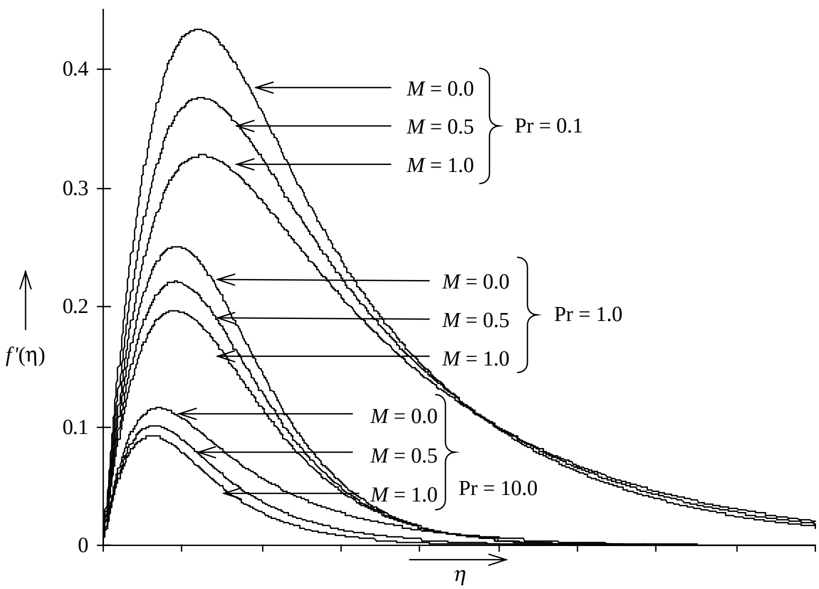

Figure 1.
Velocity distribution versus η when $\gamma = 0.0$, $\varepsilon = 0.0$ and $S = 0.0$

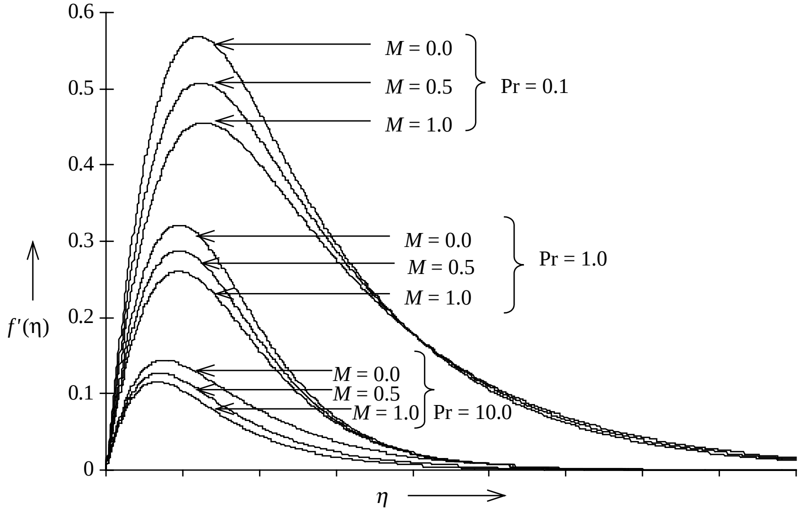

Figure 2.
Velocity distribution versus η when $\gamma = 0.0$, $\varepsilon = 0.0$ and $S = 1.0$

when magnetic intensity increases. Further with the increase in Prandtl number, the value of η at which maxima of velocity profiles is reached, shifts towards the plate (Crepeau and Clarksean, 1997).

Figure 3 shows the affect of changing magnetic field intensity on temperature profiles at different Prandtl number when $S = 1.0$. With the increase in magnetic field intensity, temperature profiles increase, while with the increase in Prandtl number the temperature profiles decrease. The figures also show that the temperature profiles smoothly decrease. It is seen from Figure 4 that when $S = 1.0$, the behaviour with respect to magnetic field is quite same when $S = 0.0$, however at $Pr = 0.1$ the

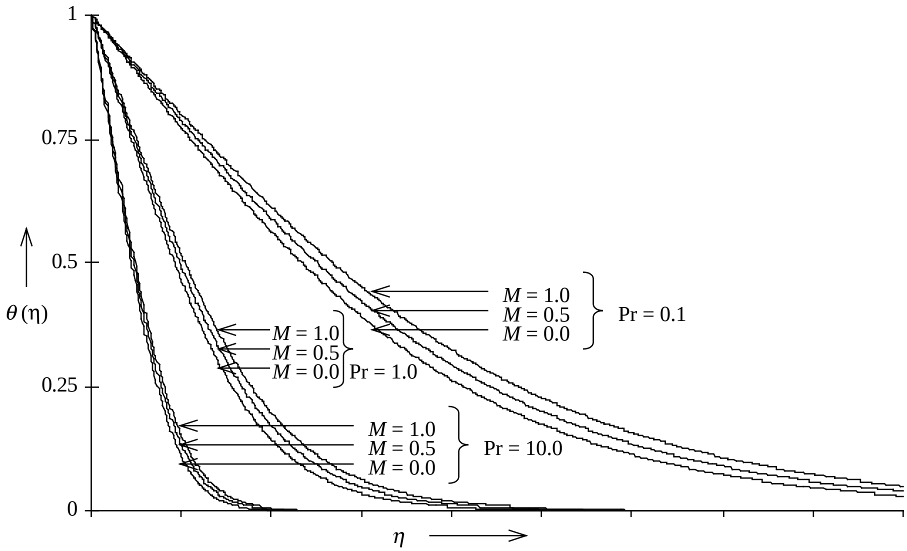

Figure 3.
Temperature distribution versus η when $\gamma = 0.0$, $\varepsilon = 0.0$ and $S = 0.0$

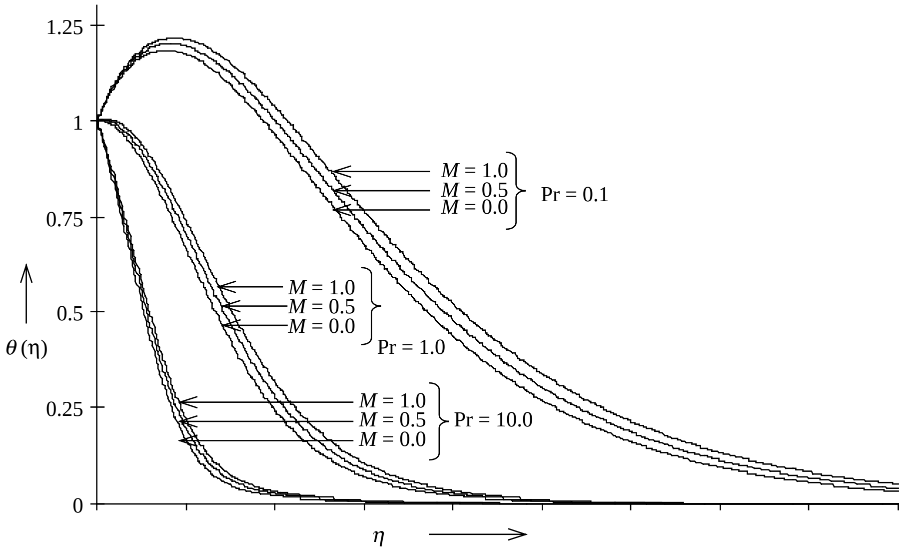

Figure 4.
Temperature distribution versus η when $\gamma = 0.0$, $\varepsilon = 0.0$ and $S = 1.0$

temperature profiles reach the value greater than 1, hence the fluid temperature exceeds the plate temperature. It is also noted that the decrease in temperature profiles near the plate is comparatively less in the case of heat generation.

Figures 5 and 6 show the comparative study of velocity profiles and temperature profiles when $S = 0.0$ and $S = 1.0$. The velocity profiles for $S = 1.0$ are higher, which

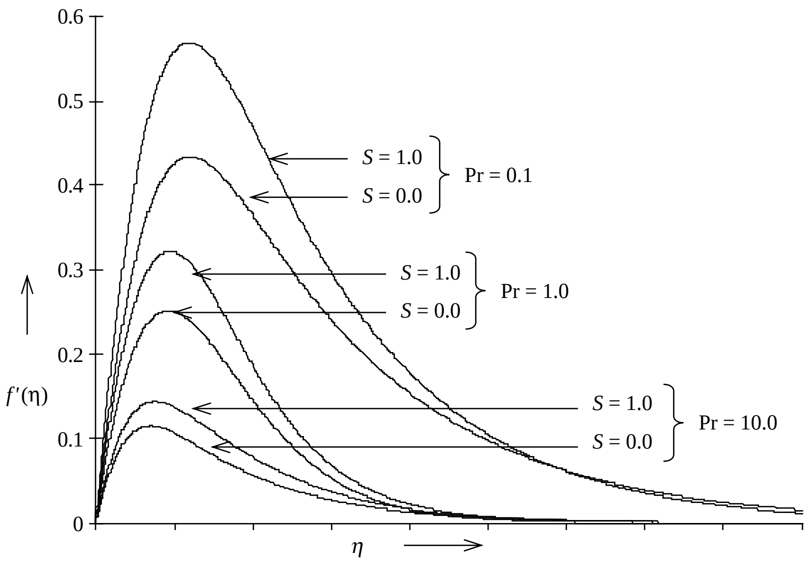

Figure 5.
Velocity distribution versus η when $\gamma = 0.0$, $\varepsilon = 0.0$ and $M = 0.0$

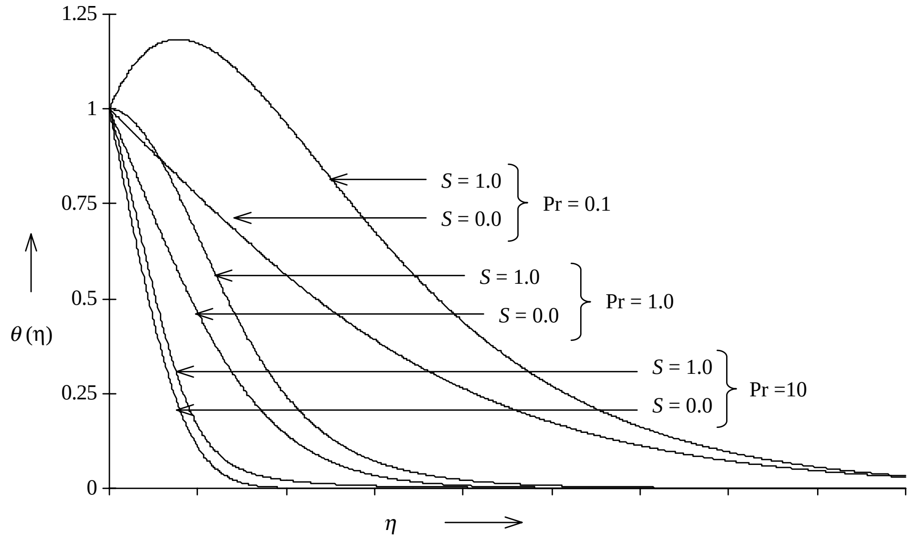

Figure 6. Temperature distribution versus η when $\gamma = 0.0$, $\varepsilon = 0.0$ and $M = 0.0$

is true, because heat generation assists buoyancy factor. The temperature profiles are also higher in case $S = 1.0$ and affect is more pronounced at lower Prandtl number.

It is observed from Figure 7 that with the increase in the parameter ε, velocity profiles increase while Figure 8 depicts that with the increase in γ results in lowering of velocity profiles near the plate when $S = 0.0$ and $Pr = 1.0$. It is seen form Figure 8 that the value of η at which maximum value of velocity profiles occurs, shifts slightly towards the plate with decrease in value of γ. Figures 9 and 10 show the velocity

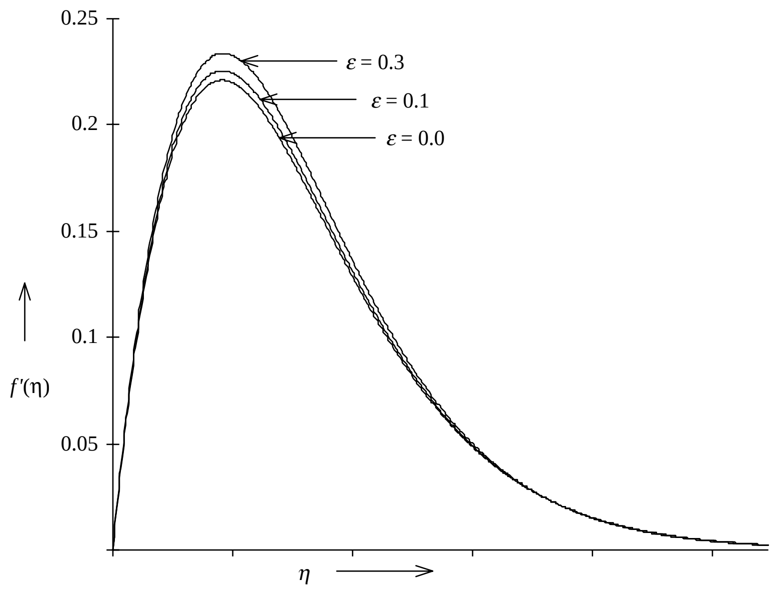

Figure 7. Velocity distribution versus η when $\gamma = 0.0$, $M = 0.5$, $Pr = 1.0$ and $S = 0.0$

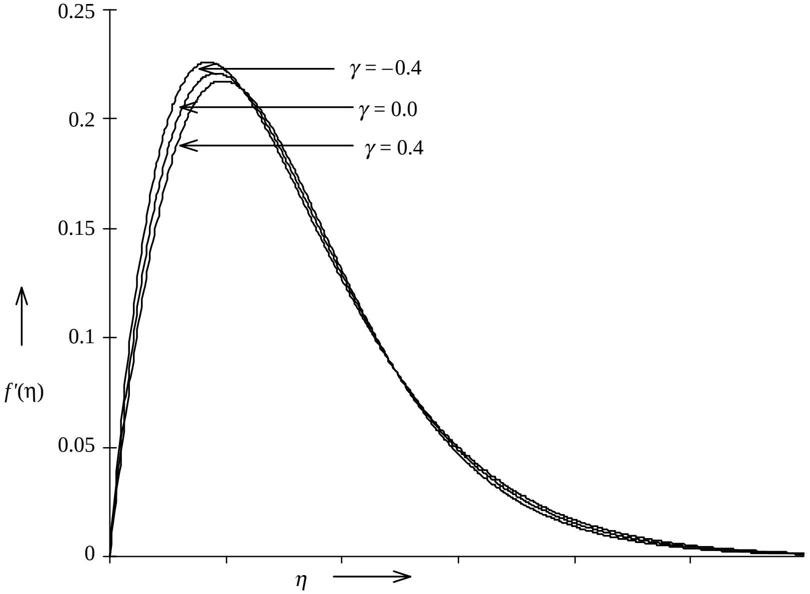

Figure 8.
Velocity distribution versus η when $M = 0.5$, $\varepsilon = 0.0$, $Pr = 1.0$ and $S = 0.0$

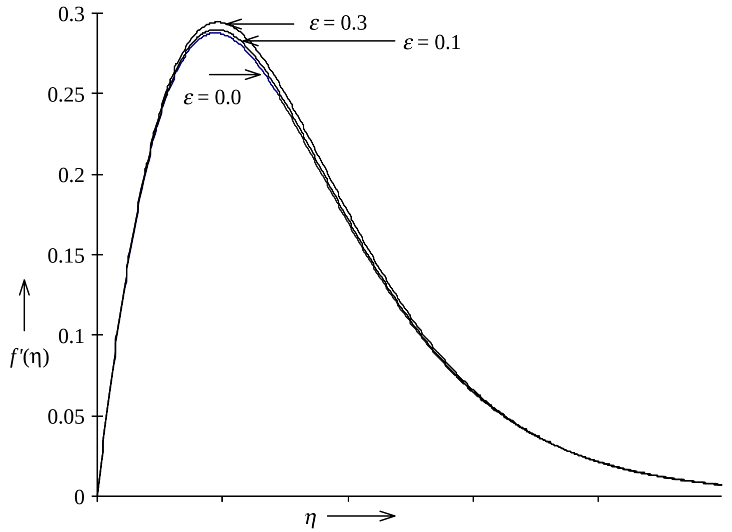

Figure 9.
Velocity distribution versus η when $\gamma = 0.0$, $M = 0.5$, $Pr = 1.0$ and $S = 1.0$

profiles and temperature profiles when $S = 1.0$, the behaviour is same as in the case $S = 0.0$ and $Pr = 1.0$. Comparative study of Figures 7-10 show that affect of ε is more pronounced when $S = 0.0$, while opposite for γ.

Figures 11 and 12 show the effect of variation of ε and γ on the temperature profiles when $S = 0.0$. With the increase in ε and γ, temperature profiles increase, a similar characteristics is observed in case when $S = 1.0$ as can be seen from Figures 13 and 14.

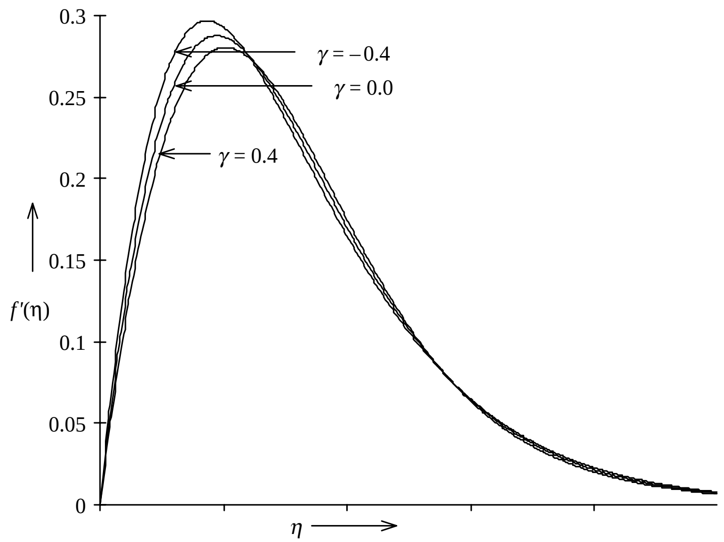

Figure 10.
Velocity distribution versus η when $\varepsilon = 0.0$, $M = 0.5$, $Pr = 1.0$ and $S = 1.0$

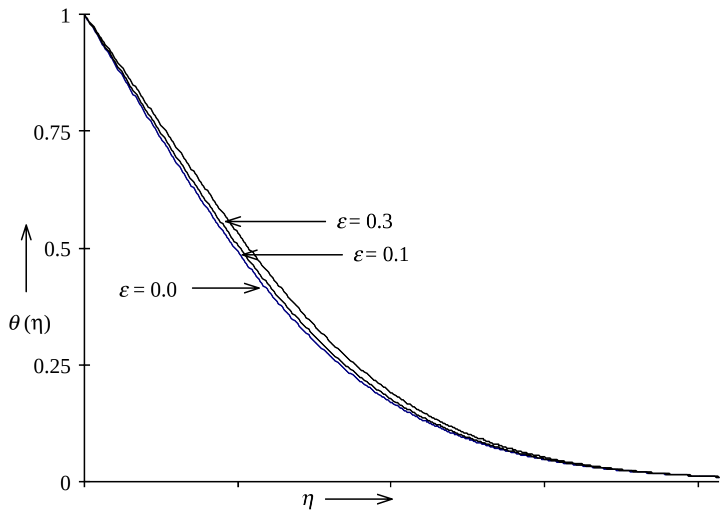

Figure 11.
Temperature distribution versus η when $\gamma = 0.0$, $M = 0.5$, Pr $= 1.0$ and $S = 0.0$

8. Conclusions

- In the absence of heat generation, skin-friction and Nusselt number at the plate decrease with the increase in γ, while increase in ε leads to increase in skin-friction and decrease in Nusselt number.
- In the presence of heat generation, both skin-friction and Nusselt number at the plate decrease with the increase in γ. With the increase in ε, skin-friction increases for $Pr = 1.0$ and 10 but it decreases when $Pr = 0.1$, while Nusselt number increases for $Pr = 0.1$ and 1.0 it but decreases when $Pr = 10$.

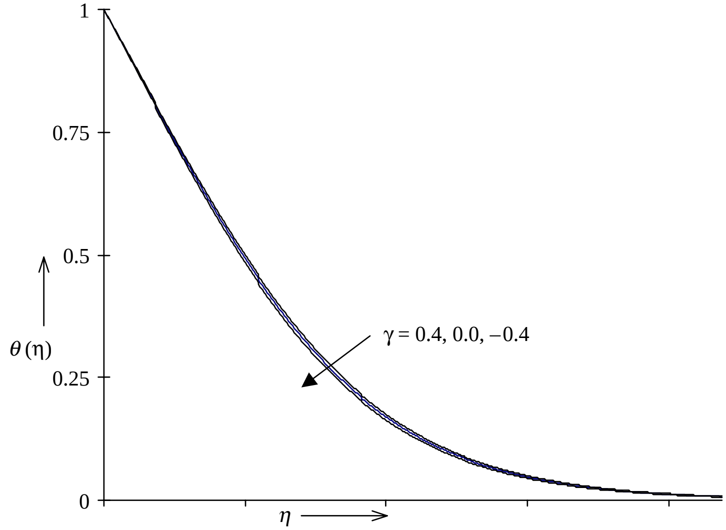

Figure 12.
Temperature distribution versus η when $\varepsilon = 0.0$, $M = 0.5$, $Pr = 1.0$ and $S = 0.0$

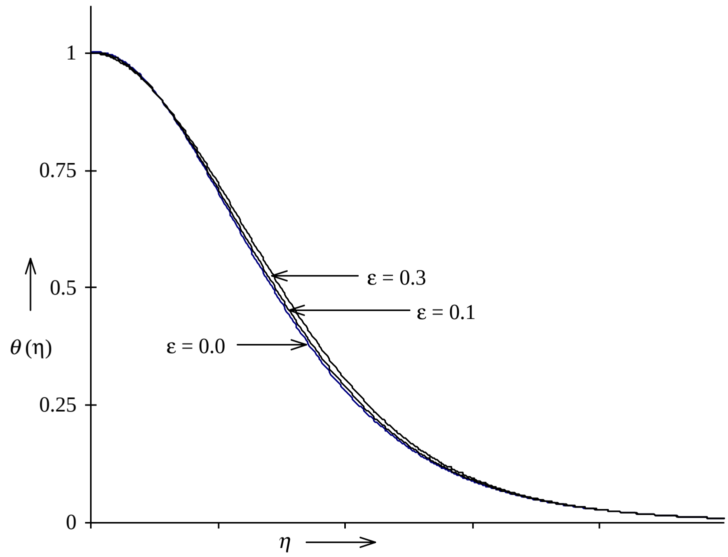

Figure 13.
Temperature distribution versus η when $\gamma = 0.0$, $M = 0.5$, $Pr = 1.0$ and $S = 1.0$

- Fluid velocity decreases with the increase in γ and it increases with the increase in ε at $Pr = 1.0$ irrespective of absence or presence of heat generation.
- Fluid temperature increases with the increase in γ and ε at $Pr = 1.0$ irrespective of presence or absence of heat generation.
- Effect of γ is more pronounced in the presence of heat generation, while opposite behaviour is true for ε.

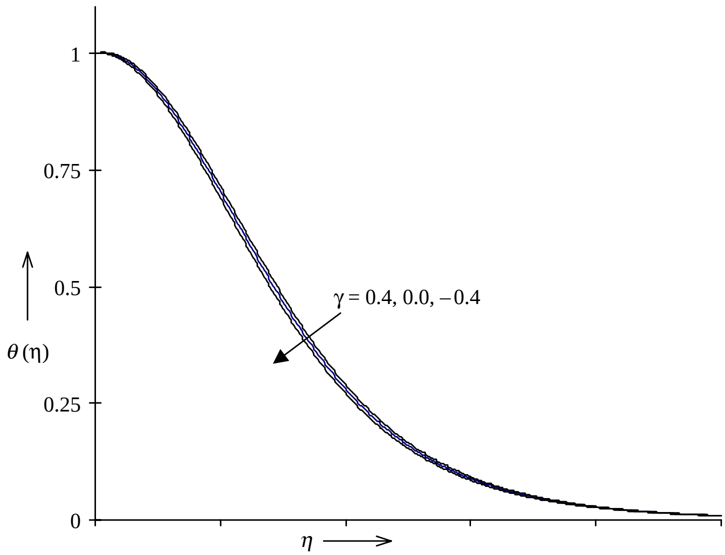

Figure 14. Temperature distribution versus η when $\varepsilon = 0.0$, $M = 0.5$, Pr $= 1.0$ and $S = 1.0$

- Fluid velocity decreases with the increase in magnetic parameter or Prandtl number.
- Fluid temperature increases with the increase in magnetic parameter.
- Velocity and temperature profiles increase due to increase in heat generation parameter.

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